$$\tau = U_0 t/b \,;\, \beta = U/U_0$$

$$I_3 = \frac{\zeta^{1-2n}}{(1 + [1 - \zeta^2]^{1/2})^{2-2n}}$$

$$I_1 = \int_0^1 d\zeta/I_3 \,;\, I_2 = \int_0^\zeta d\zeta/I_3$$

$$(\dot{\ }) = \frac{d}{d\tau}(\ ) \,;\, (\ )' = \frac{d}{d\alpha}(\ )$$

and $\Gamma^*(\zeta)$ is defined by the equation

$$\Gamma(\zeta) = \left(\frac{K}{U}\right)^2 \frac{1}{\pi} \frac{d\alpha}{dt} \Gamma^*(\zeta)$$

The Moment Equation

For dynamic equilibrium of the parachute system the total moment about any axis must be equal to zero. On equating the moment about an axis through the apex of the wedge, firstly due to aerodynamic pressure differential across the wedge surface, and secondly due to closing moment arising from rigging line tension, we have

$$\int_s psds - b \sin(\alpha + \delta) \sin\alpha/\cos\beta \int_s pds = 0 \quad (14)$$

Substituting Eq. (13) in (14) and collecting like terms we obtain the following nonlinear second-order differential equation

$$C_1\ddot{\alpha} + C_2\dot{\alpha}^2 + C_3\beta\dot{\alpha} + C_4\dot{\beta} + C_5 = 0 \quad (15)$$

where the coefficients C_1, C_2, C_3, C_4, and C_5 are all functions of wedge angle α and the ratio (L/b). The variation of these coefficients for $L/b = 2.0$ is shown in Fig. 3.

It can be seen that Eq. (15) is nonlinear in α and its time derivatives. Knowing the initial conditions [say $\alpha(0)$ and $\dot{\alpha}(0)$ are given] the equation is numerically evaluated by using the modified Range-Kutta method of Mersion.[6]

Conclusions

The aim of the study is to develop concrete ideas on how the parachute inflation process can be analyzed using analytical techniques. With the structural model shown in Fig. 1, the unsteady pressure distribution on the decelerating, inflating canopy surface is established. Then the moment equation yields the differential Eq. (15). This equation is solved for infinite mass case (i.e., $\dot{\beta} = 0$). From Fig. 4, it is clear the solution of the equation for α as a function of time, effectively replaces the filling-time notions inherent in the filling time theory. The equation basically describes a damped oscillation system in which two initial conditions [given $\alpha(0)$ and $\dot{\alpha}(0)$] will determine changes and palpatations in α as a function of time. The form of equation admits the over-inflation phenomenon followed by small damped oscillations in α until a steady inflated value $\bar{\alpha}$ is reached.

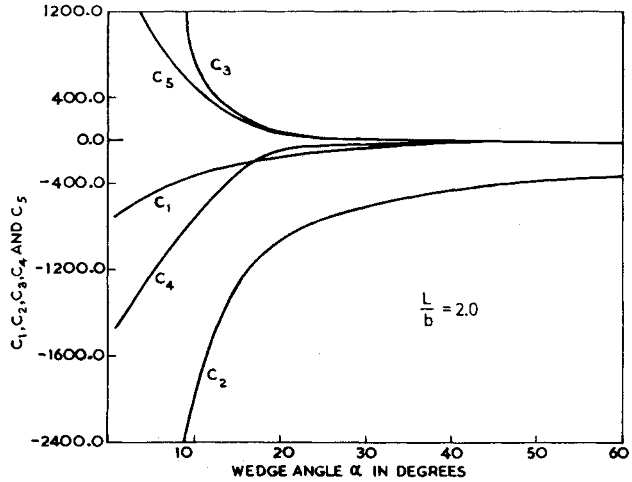

Fig. 3 Coefficients C_1, C_2, C_3, C_4, and C_5.

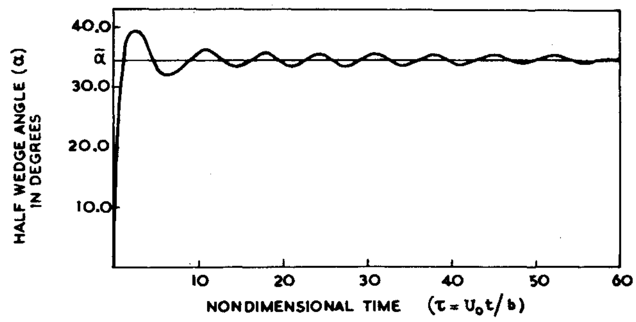

Fig. 4 Variation of wedge angle with time.

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Minimum Noise Climbout Trajectories of a VTOL Aircraft

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Nomenclature

x, h	= flight path coordinates
XP	= abscissa of observer point
R	= distance aircraft-observer
ϕ	= angle of radiation
PNL	= perceived noise level
EPNL	= effective preceived noise level

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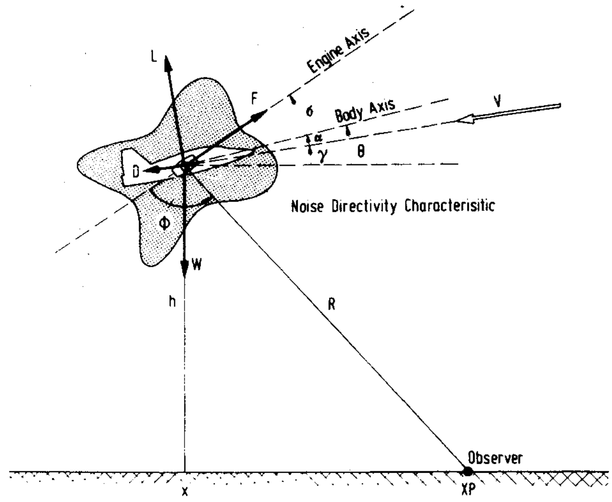

Fig. 1 Flight mechanic and noise parameters.

CF = cost function
γ_o = optimization parameter = flight path angle of a straight path segment
$\Delta\gamma$ = variation interval

Subscripts

B, E = beginning and end of the flight maneuver

Introduction

NOISE annoyance to the population in the vicinity of airports from aircraft taking off and landing has grown to such a degree that it has become necessary to investigate how this negative effect of air traffic can be reduced. Efforts may be concentrated on the two fields of construction of more quiet engines and determination of suitable takeoff and landing maneuvers by which the noise annoyance in definite areas will be kept as low as possible.

The present paper deals with the computation of minimum noise climbout trajectories. For this, the determination of an appropriate measure for noise annoyance is important as well as the choice of a suitable optimization procedure. The optimization procedure must be selected so that it is possible to take flight mechanic restrictions into account without great efforts.

In this study, calculations are performed for a special VTOL aircraft. In determining minimum noise trajectories only such noise sensitive areas are considered which are located along the ground track of the climbout path.

Analysis

The aircraft noise perceived by an observer depends on the distance aircraft-observer, R, the noise intensity of engines, that is the magnitude of thrust, F, and the angle of radiation, ϕ (Fig. 1). The dependence on angle ϕ is based on the fact that the engines have noise directivity characteristics. According to Ref. 1, the perceived noise level PNL, in units of PNdb, can be given in the simplified form

$$\text{PNL} = \text{PNL}_0(\phi,R_0,F_0) - 50\log\frac{F_0}{F} - 20\log\frac{R}{R_0} - \beta(R-R_0) \quad (1)$$

Here, PNL_o is the basic noise which has been measured for a definite aircraft as a function of angle ϕ at a reference distance R_o and a reference thrust F_o. The following terms indicate the noise attenuation due to thrust reduction, spherical spreading, and atmospheric absorption. Thrust reduction also may cause a modification of the directivity characteristic PNL_o. This fact must be brought into account in special cases. The atmospheric absorption rate β depends on the atmospheric conditions and the frequency. Assuming average values for temperature and humidity and a mean frequency of 500 Hz, β results to be about 0.003 db/m.[2]

For noise annoyance, not only the magnitude of noise plays a part, but also its duration. The effective perceived noise level EPNL, in units of EPNdb, as defined in Part 36 of the FAR,[2] takes both influences into account:

$$\text{EPNL} = 10\log\left\{\frac{1}{10}\int_{t_1}^{t_2} 10^{\text{PNL}(t)/10}dt\right\} \quad (2)$$

In this, $[t_1,t_2]$ signifies that time interval in which $\text{PNL}(t) \geqq \text{PNL}_{\max} - 10$ is valid. Since, when varying the flight path, t_1 and t_2 are not known in advance, EPNL in this form cannot be used as cost function for an optimization. In order to get a suitable cost function one has to replace $[t_1,t_2]$ by the total time interval $[t_B,t_E]$ of the flight maneuver.

When considering a region consisting of several observer points P_i $(i = 1,\ldots,m)$ with different noise sensitivity, the cost function may be extended in the following way: The local effective perceived noise levels EPNL_i are increased by an amount K_i compared with the least noise sensitive point and then averaged antilogarithmically. With this

$$\text{CF} = 10\log\left\{\frac{1}{10}\int_{t_B}^{t_E}\frac{1}{m}\sum_{i=1}^{m}10^{(\text{PNL}_i(t)+K_i)/10}dt\right\} \quad (3)$$

results.

The value of cost function CF depends on the flight maneuver performed. When considering a VTOL aircraft with a tiltable group of engines the trajectory of which is approximately assumed to be piecewisely straight, its longitudinal motion can be described by the following equations:

$$m\frac{dV}{dt} = F\cos(\sigma+\alpha) - \frac{1}{2}\rho SV^2C_D(\alpha) - W\sin\gamma_0 \quad (4)$$

$$0 = F\sin(\sigma+\alpha) + \frac{1}{2}\rho SV^2C_L(\alpha) - W\cos\gamma_0 \quad (5)$$

$$(dx/dt) = V\cos\gamma_0 \quad (6)$$

$$h = h_0 + (x - x_0)\tan\gamma_0 \quad (7)$$

where m = mass of aircraft, V = airspeed, σ = thrust vector angle, α = angle of attack, ρ = air density, S = wing area, $C_D(\alpha)$ = drag coefficient, $C_L(\alpha)$ = lift coefficient, W = weight, γ_o = flight path angle of a straight path segment, x and h = flight path coordinates, and x_o and h_o = initial coordinates of a straight path segment (see Fig. 1). The thrust F, thrust vector angle σ and, with regard to passenger comfort, also the angle of pitch $\theta = \gamma_o + \alpha$ shall be constrained.

The motion of aircraft is controllable by the variables F, σ, and α. These control variables are, however, connected with one another by the transcendent Eq. (5). If V, γ_o and F are prescribed, then σ and α are to be chosen so that Eq. (5) is satisfied at any time taking the above mentioned constraints into account. At first one equates $\alpha = \alpha_{\max} = \theta_{\max} - \gamma_o$ and tries to satisfy the equation by the choice of σ. If this is not possible, one tries to succeed by reducing α. If this is not possible, then the path segment is not flyable with the prescribed values of V, γ_o, and F. This case occurs, for example, if one tries to fly a steep path segment at high speed. Because of the θ restriction $\alpha = \theta - \gamma_o$ will be negative. If the speed V is high enough, then the lift becomes negative to such a degree that the right hand side of Eq. (5) is negative for all values of F and σ.

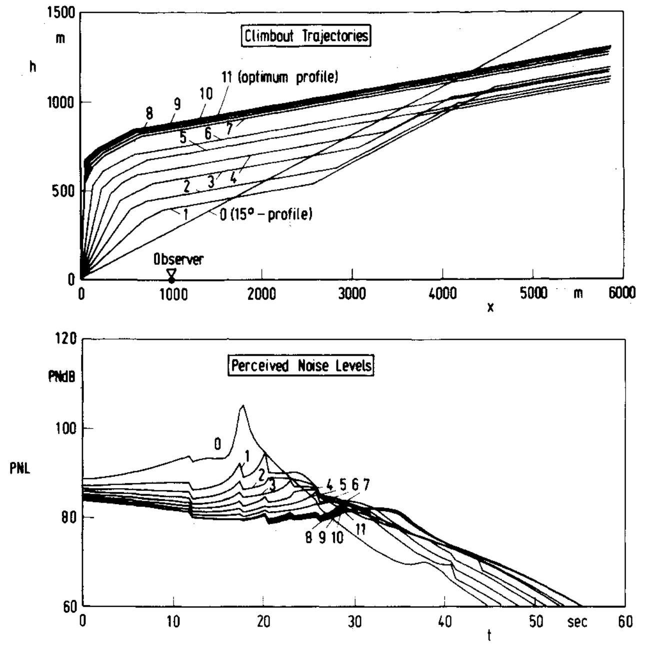

Fig. 2 Climbout trajectories and the corresponding perceived noise levels PNL for different extremization sequences.

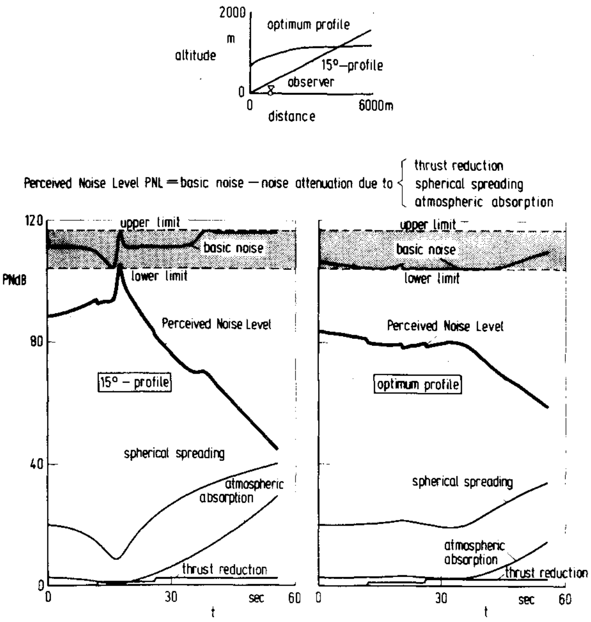

Fig. 3 Perceived noise level PNL and its components of a 15°-profile and of the optimum profile.

The optimization problem can now be formulated as follows: from the manifold of flyable, piecewisely straight climbout paths, that path shall be chosen that makes the cost function, Eq. (3), a minimum. Assuming the observer points P_i to be located along the ground track of the climbout path and denoting their abscissa with XP_i, the distances R_i and the angles of radiation ϕ_i needed for the calculation of the local perceived noise levels $PNL_i(t)$ can be expressed by the flight mechanic variables (see Fig. 1):

$$R_i = (h^2 + (x - XP_i)^2)^{1/2} \tag{8}$$

$$\phi_i = (\pi/2) - \arctan[((x - XP_i)/h)] - \sigma - \theta \tag{9}$$

The variables V, σ, α, x, and h can be determined from Eq. (4)-(7). If a thrust program $F(t)$ is given, then only the variable γ_o remains free. It is obvious to use γ_o as optimization parameter. This means that the gradients of the individual straight path segments have to be varied so that the cost function (3) becomes a minimum. For the flight maneuver, the following boundary conditions are assumed: at the time $t = t_B$, the flight path coordinates x_B and h_B and airspeed V_B are given; at $t = t_E$ all variables are left free.

Because of the assumption of piecewise straight trajectories, the problem was discretized with reference to the optimization parameter γ_o. This enables the solution of the optimization problem by means of Bellman's method of dynamic programing.[3,4] For computation a variant of this method,[5] saving time and storage capacity, is used.

Noise optimization is performed for a VTOL aircraft with a thrust/weight ratio of $F_{max}/W = 1.26$. In the first case, a single observer point at $XP = 1000$ m is considered. Starting from a 15°-trajectory and computing with a variation interval of $\Delta\gamma = 10°$, the optimal solution is found after 11 extremization sequences. Figure 2 shows the trajectories and the corresponding perceived noise levels PNL for the different extremization sequences. The effective perceived noise level EPNL was reduced from 96.5 to 87.2 EPNdb. When continuing the optimization with the refined variation interval $\Delta\gamma = 2°$, a further reduction of EPNL to 86.6 EPNdb results.

In order to recognize which factors are decisive for noise minimization, the various noise components of the 15°-trajectory and of the optimum trajectory calculated with $\Delta\gamma = 2°$ are represented in Fig. 3. In the case of the optimum trajectory, the basic noise runs along its lower boundary for the most time. That means that the engines are adjusted so that the observer is in the direction of minimum noise radiation. Beyond that, the minimum values of the spherical spreading and atmospheric absorption are greater than in the case of the 15°-trajectory, corresponding to the larger distance when flying over the observer. The optimum trajectory is thus characterized by a distance to the observer as large as possible and an engine position as favorable as possible.

As the second case, noise optimization is performed with reference to a region consisting of four observer points at $XP = 1000$, 1500, 2000, and 2500 m with different noise sensitivity. Noise sensitivity is expressed by the quantities K_i, about which the noise levels are increased in the different points. For the four points in the above mentioned sequence, K_i is chosen to be 0, 5.5, 10 and 13 db. Figure 4 shows a 15°-trajectory and the optimum trajectory together with the corresponding effective perceived noise levels EPNL. One can recognize here that the EPNL-values in all observer points are considerably reduced due to the optimization and adapt themselves to the prescribed noise sensitivity.

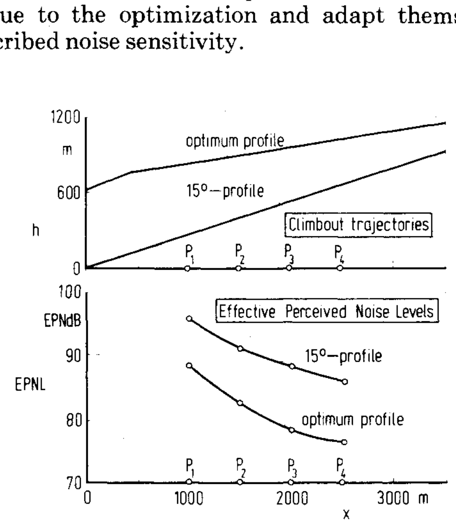

Fig. 4 15°-profile and optimum profile and the corresponding effective perceived noise levels EPNL.

Conclusions

The aim of the present study is to investigate how far aircraft noise can be reduced by special takeoff procedures. This problem requires the application of optimization methods. For this, one has to define an appropriate cost function and to choose an optimization method adapted to the problem. As a cost function, a modification and/or extension of the effective perceived noise level EPNL is used. Because of the assumption of piecewise straight flight paths, optimization can be made by means of dynamic programing. For computation, a variant of this method, which saves time and storage capacity, is employed. The effectiveness of optimization is shown in the case of a special VTOL aircraft. The result is that the optimum trajectory is essentially influenced by the noise directivity characteristics of the engines. More details are given in Ref. 6.

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